Permittivity and Permeability Tensors for a Paraboloidal Invisibility Cloaking Shell

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***Abstract*— Invisibility or stealth in aerospace is a foremost requirement which can be achieved through various techniques such as shaping, radar absorbing materials or adaptive techniques. The constraints in all these techniques can be put together to move forward in the research of achieving the optimal version of electromagnetic (EM) stealth property. Invisibility cloaking is amongst one of them, in which, the EM waves can be controlled within the cloaking shell by introducing a prescribed spatial variation in the constitutive parameters. In this paper, analytical expressions of the permittivity and permeability tensors have been derived for General Paraboloid of Revolution (GPOR) structure and the final expressions along with simulation results are reported. These analytical expressions will help the EM designers to analyze and design of cloaking shells of paraboloidal shape and their hybrids which can lead to design of cloaking shells of arbitrary shapes.**

Keywords—Permittivity and Permeability Tensors, Invisibility cloak, GPOR, Spatial metric tensors

I. INTRODUCTION

Invisibility cloak design is a dream coming true for the researchers because of the developments in the metamaterial science and technology. In this method, the electromagnetic (EM) waves are controlled within a material by introducing a prescribed spatial variation within the constitutive parameters [1]. The most challenging task in design of invisibility cloaking shell is the calculation of permittivity and permeability tensors at each and every layer in a desired manner.

The basic parameters such as the dielectric constant and the permeability are *a priori* requirement for propagation analysis of EM waves within the matter [2]. For anisotropic random media [3] these fundamental parameters are expressed as tensors.

The invisibility cloaking structure is dependent on the type of object to be cloaked [4]. Hence, it is important to calculate the permeability and permittivity tensors in all the directions *w.r.t.* the quadric surfaces. In this paper, closed form analytical expressions for permeability and permittivity tensors for a cloaking shell of GPOR shape is provided. Towards this, the parametric equations of GPOR are used and the corresponding spatial metric along with effective geometry is derived. Due to space constraints the final expressions of effective geometry and the derived analytical expression of permittivity and permeability tensors of a GPOR shaped cloaking shell is provided.

II. PERMITTIVITY TENSOR FOR CLOAKING SHELL

General paraboloid of revolution and its hybrids constitute an important class of structures within aerospace domain due to their applications to radomes, space module etc.

The analytical formulation of permittivity and permeability tensors starts at the transformation optics of Rectangular coordinate to curvilinear coordinate system. The Rectangular coordinates (x, y, z) can related to the Curvilinear Coordinates (u,v,w), for the GPOR with the important basis variables (u,v). The parametric equations of a GPOR may be expressed as [5]

$$x = au\cos\phi; \quad y = au\sin\phi; \qquad z = -u^2; \tag{1}$$

where, a is the shaping parameter obtained by holding the third basis variable w constant, u varies from zero to infinity and Φ is the angle varying from 0° to 360°. Figure 1 shows the boundary of a GPOR cloaking shell.

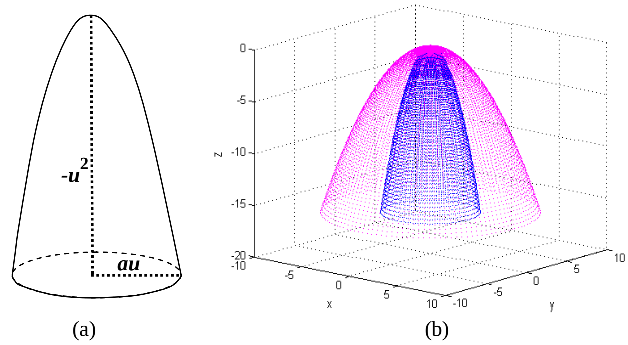

Fig. 1. (a) Schematic of GPOR (b) Boundary of a GPOR cloaking shell

Using the parametric Eq. (1), the position vector is expressed as:

$$\vec{r} = (au\cos\phi)\hat{i} + (au\sin\phi)\hat{j} - u^2\hat{k} \tag{2}$$

The orthogonal vectors of the paraboloidal cloaking surface are used for derivation of the effective geometry in curvilinear coordinate system.

The effective geometry corresponding to the bi-anisotropic medium is derived and the final expression is given as:

$$g^{ij} = \begin{pmatrix} \left(\frac{a_2 - a_1}{a_2}\right)^2 \frac{1}{u'^2} & 0 & 0 \\ 0 & \frac{1}{a'^2 + 4u'^2} & 0 \\ 0 & 0 & \frac{1}{a'^2 u'^2} \end{pmatrix} \tag{3}$$

The volume element of the space is given as

$$\sqrt{g} = \frac{a_2}{a_2 - a_1} a' u'^2 \sqrt{\left(a'^2 + 4u'^2\right)} \tag{4}$$

Hence, the permittivity tensor which is same as permeability tensor is derived using Eq. (2) through Eq. (4) as:

$$\varepsilon^{ij} = \mu^{ij} = \pm \frac{\sqrt{g}}{\sqrt{r}} g^{ij}$$

$$\varepsilon^{ij} = \begin{pmatrix} \left(\frac{a_2 - a_1}{a_2}\right) a \sqrt{(a'^2 + 4u'^2)} & 0 & 0 \\ 0 & \left(\frac{a_2}{a_2 - a_1}\right) \frac{a' u'^2}{\sqrt{(a'^2 + 4u'^2)}} & 0 \\ 0 & 0 & \left(\frac{a_2}{a_2 - a_1}\right) \frac{\sqrt{(a'^2 + 4u'^2)}}{a'} \end{pmatrix} \frac{1}{\sqrt{(a^2 + 4u^2)\, a u^2}} \tag{5}$$

Here, it is pointed out that, the because of space constraints the final analytical expressions for permittivity and permeability tensors are provided.

III. SIMULATION RESULTS

In the previous section the closed form analytical expressions for permittivity tensor is provided. The permittivity tensor derived is used for simulation of typical GPOR cloaking shell having dimensions, a_1=0.1, a_2=0.2 and u as $0 \leq u \leq 4$. To hide an object of dimension $0 < a \leq a_1$, the permittivity tensors in the uu and $\Phi\Phi$ directions are given in Figure 2 through 4.

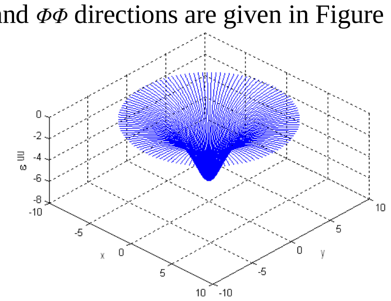

Fig. 2. Permittivity tensor ε^u distribution inside the invisibility cloaking shell of GPOR shape.

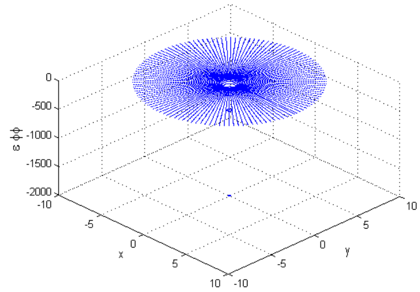

Fig. 3. Permittivity tensor ε^{Φ} distribution inside the invisibility cloaking shell of GPOR shape.

It can be observed from the simulation results that for getting invisibility one has to go for zero refractive index and negative refractive index materials for which metamaterial structures should be designed.

IV. CONCLUSION

Analytical expressions for calculation of permittivity and permeability tensors for a GPOR shaped cloaking shells are provided in this paper. As the analysis is done using the parametric equation of GPOR, by changing the shaping parameter, paraboloidal cloaking shells of varying curvature can be designed. This work will provide a solution to the challenging task of designing invisibility cloaking shell. These derivations can be used for design of cloaking devices of various hybrid structures.

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